

Mechanics M3 Mark scheme

Question	Scheme	Marks
1	(30° or θ for the first 3 lines)	
	$R \sin 30^\circ = mg$	M1 A1
	$R \cos 30^\circ = m(r \cos 30^\circ) \omega^2$	M1 A1 A1
	$\omega^2 = \frac{R}{mr} = \frac{g}{r \sin 30}$	DM1
	$\omega = \sqrt{\frac{2g}{r}}$	A1
	Time = $\frac{2\pi}{\omega} = 2\pi \sqrt{\frac{r}{2g}} = \pi \sqrt{\frac{2r}{g}}$ *	A1 cso
		(8)
	Alternative:	
	Resolve perpendicular to the reaction:	
	$mg \cos 30 = m \times rad \times \omega^2 \cos 60$	M2 A1 (LHS) A1 (RHS)
	$= mr \cos 30 \omega^2 \cos 60$	A1
	Obtain ω	M1 A1
	Correct time	A1
		(8)
(8 marks)		
Notes:		
M1: Resolving vertically 30° or θ A1: Correct equation 30° or θ M1: Attempting an equation of motion along the radius, acceleration in either form 30° or θ Allow with r for radius. A1: LHS correct 30° or θ A1: RHS correct, 30° or θ but not r for radius. DM1: Obtaining an expression for ω^2 or for v^2 and the length of the path 30° or θ Dependent on both previous M marks. A1: Correct expression for ω Must have the numerical value for the trig function now. A1cso: Deducing the GIVEN answer.		

Question	Scheme	Marks
2(a)	$F = \frac{K}{x^2}$	
	$x = R \Rightarrow F = mg \quad \therefore mg = \frac{K}{R^2}$	M1
	$K = mgR^2 *$	A1
		(2)
(b)	$\frac{mgR^2}{x^2} = -mv \frac{dv}{dx}$	M1
	$g \int \frac{R^2}{x^2} dx = - \int v dv$	
	$-g \frac{R^2}{x} = -\frac{1}{2}v^2 \quad (+c)$	dM1 A1ft
	$x = 3R, v = V \Rightarrow -g \frac{R^2}{3R} = -\frac{1}{2}V^2 + c$	M1
	$c = -\frac{Rg}{3} + \frac{1}{2}V^2$	A1
	$x = R \Rightarrow \frac{1}{2}v^2 = -\frac{Rg}{3} + \frac{1}{2}V^2 + g \frac{R^2}{R}$	M1
	$v^2 = V^2 + \frac{4Rg}{3}$	
	$v = \sqrt{V^2 + \frac{4Rg}{3}}$	A1 cso
		(7)
(9 marks)		
Notes:		
(a)		
M1: Setting $F = mg$ and $x = R$		
A1: Deducing the GIVEN answer		
(b)		
M1: Attempting an equation of motion with acceleration in the form $v \frac{dv}{dx}$. The minus sign may be missing.		
dM1: Attempting the integration.		
A1ft: Correct integration, follow through on a missing minus sign from line 1, constant of integration may be missing.		
M1: Substituting $x = 3R, v = V$ to obtain an equation for c		
A1: Correct expression for c .		
M1: Substituting $x = R$ and their expression for c .		
A1: Correct expression for v , any equivalent form.		

Question	Scheme	Marks
3(a)	$\frac{dv}{dt} = -2(t+4)^{-\frac{1}{2}}$	M1
	$v = -\int 2(t+4)^{-\frac{1}{2}} dt$	
	$v = -4(t+4)^{\frac{1}{2}} \quad (+c)$	dM1 A1
	$t = 0, v = 8 \Rightarrow c = 16$	M1
	$v = 16 - 4(t+4)^{\frac{1}{2}} \quad (\text{m s}^{-1}) \quad *$	A1 cso
		(5)
(b)	$v = 0 \quad 16 = 4(t+4)^{\frac{1}{2}}$	M1
	$16 = t + 4 \quad t = 12$	A1
	$x = 4 \int \left(4 - (t+4)^{\frac{1}{2}} \right) dt$	
	$x = 4 \left(4t - \frac{2}{3}(t+4)^{\frac{3}{2}} \right) \quad (+d)$	M1 A1
	$t = 0, \quad x = 0 \quad d = 4 \times \frac{2}{3} \times 4^{\frac{3}{2}} = \frac{64}{3} \quad \text{oe}$	A1
	$t = 12 \quad x = 4 \left(4 \times 12 - \frac{2}{3} \times 16^{\frac{3}{2}} \right) + \frac{64}{3} = 42 \frac{2}{3} \quad (\text{m}) \quad \text{oe eg 43 or better}$	dM1 A1
		(7)
	(12 marks)	
Notes:		
(a)		
M1: Attempting an expression for the acceleration in the form $\frac{dv}{dt}$; minus may be omitted.		
DM1: Attempting the integration		
A1: Correct integration, constant of integration may be omitted (no ft)		
M1: Using the initial conditions to obtain a value for the constant of integration		
A1: cso. Substitute the value of c and obtain the final GIVEN answer		
(b)		
M1: Setting the given expression for v equal to 0		
A1: Solving to get $t = 12$		
M1: Setting $v = \frac{dx}{dt}$ and attempting the integration wrt t . At least one term must clearly be integrated.		
A1: Correct integration, constant may be omitted.		

Question 3 notes *continued*

M1: Substituting $t = 0$, $x = 0$ and obtaining the correct value of d . Any equivalent number, inc decimals.

dM1: Substituting their value for t and obtaining a value for the required distance. Dependent on the second M mark.

A1: Correct final answer, any equivalent form.

Question	Scheme	Marks
4(a)	Energy to top: $\frac{1}{2} \times 3m \times u^2 - \frac{1}{2} \times 3mv^2 = 3mga$	M1 A1
	NL2 at top: $T + 3mg = 3m \frac{v^2}{a}$	M1 A1
	$T = 3m \frac{u^2}{a} - 6mg - 3mg$	dM1
	$T \geq 0 \Rightarrow \frac{u^2}{a} \geq 3g$	M1
	$u^2 \geq 3ag$	A1 cso
		(7)
(b)	Tension at bottom:	
	$\frac{1}{2} \times 3m \times V^2 - \frac{1}{2} \times 3mu^2 = 3mga$	M1
	$T_{\max} - 3mg = 3m \frac{V^2}{a}$	M1
	$T_{\max} = 3mg + 6mg + 3m \frac{u^2}{a}$	A1
	$T_{\min} = 3m \frac{u^2}{a} - 9mg$	
	$9mg + 3m \frac{u^2}{a} = 3 \left(3m \frac{u^2}{a} - 9mg \right)$	dM1
	$u^2 = 6ag$ *	A1 cso
		(5)
(12 marks)		
Notes:		
(a) M1: Attempting an energy equation, can be to a general point for this mark. Mass can be missing but use of $v^2 = u^2 + 2as$ scores M0 A1: Correct equation from A to the top. M1: Attempting an equation of motion along the radius at the top, acceleration in either form. A1: Correct equation, acceleration in form $\frac{v^2}{r}$ dM1: Eliminate v^2 to obtain an expression for T dependent on both previous M marks. M1: Use $T \geq 0$ at top to obtain an inequality connecting a, g and u A1: Re-arrange to obtain the GIVEN answer.		

Question 4 notes *continued*

(b)

M1: Attempting an energy equation to the bottom, maybe from A or from the top.

M1: Attempting an equation of motion along the radius at the bottom.

A1: Correct expression for the max tension.

dM1: Forming an equation connecting *their* tension at the top with *their* tension at the bottom. If the 3 is multiplying the wrong tension this mark can still be gained. Dependent on both previous M marks.

A1: **cso.** Obtaining the GIVEN answer.

Question	Scheme	Marks
5(a)	$T = \frac{20e}{2} = \frac{15(1.8-e)}{1.2}$	M1A1
	$10e \times 1.2 = 15(1.8-e)$	
	$e = 1$	A1
	$AO = 3\text{ m}$ *	A1cso
		(4)
(b)	$0.5\ddot{x} = \frac{20(1-x)}{2} - \frac{15(0.8+x)}{1.2}$	M1 A1 A1
	$\ddot{x} = -45x \quad \therefore \text{SHM}$	A1 cso
		(4)
(c)	String becomes slack when $x = (-)0.8$ (allow wo sign due to symmetry)	B1
	$v^2 = \omega^2(a^2 - x^2)$	
	$v^2 = 45(1 - 0.8^2)$ (=16.2)	M1 A1 ft
	$v = 4.024\ldots \text{ m s}^{-1}$ (4.0 or better)	A1ft
		(4)
(d)	$\frac{1}{2} \times \frac{20y^2}{2} - \frac{1}{2} \times \frac{20 \times 1.8^2}{2} = \frac{1}{2} \times 0.5 \times 16.2$ ft on v	M1 A1 A1 ft
	$20y^2 - 64.8 = 16.2$	
	$y^2 = 4.05 \quad y = 2.012\ldots$	A1
	Distance $DB = 5 - 4.012\ldots = 0.988\ldots \text{ m}$ (accept 0.99 or better)	A1ft
	Alternative	
	$0.5a = -10(1.8 + x)$	
	$v \frac{dv}{dx} = -36 - 10x$	
	$\int v dv = -\int (36 + 10x) dx$	
	$\frac{v^2}{2} = -36x + 5x^2 + c$	M1 A1
	$x = 0, v = \frac{9\sqrt{5}}{5} \therefore c = 8.1$	A1
	Then $v = 0$ etc	M1 A1
		(5)
(17 marks)		

Question 5 continued**Notes:****(a)****M1:** Attempting to obtain and equate the tensions in the two parts of the string.**A1:** Correct equation, extension in AP or BP can be used or use OA as the unknown.**A1:** Obtaining the correct extension in either string (ext in $BP = 0.8$ m) or another useful distance.**A1:** **cso.** Obtaining the correct GIVEN answer.**(b)****M1:** Forming an equation of motion at a general point. There must be a difference of tensions, both with the variable. May have m instead of 0.5 Accel can be a .**A1 A1:** Deduct 1 for each error, m or 0.5 allowed, acceleration to be $\ddot{x}$ now.**A1:** **cso** Correct equation in the required form, with a concluding statement; m or 0.5 allowed.**Question 5 notes continued****(c)****B1:** For $x = \pm 0.8$ Need not be shown explicitly.**M1:** Using $v^2 = \omega^2 (a^2 - x^2)$ with *their* (numerical) ω and their x **A1ft:** Equation with correct numbers ft their ω **A1ft:** Correct value for v 2sf or better or exact.**(d)****M1:** Attempting an energy equation with 2 EPE terms and a KE term.**A1:** 2 correct terms may have $(1.8 + x)$ instead of y .**A1ft:** Completely correct equation, follow through their v from (c)**A1:** Correct value for distance travelled after PB became slack. $x = 0.21$ **A1ft:** Complete to the distance DB . Follow through their distance travelled after PB became slack.

Question	Scheme	Marks
6(a)	$\text{Vol} = \pi \int_0^2 (x^2 + 3)^2 dx$	M1
	$= \pi \int_0^2 (x^4 + 6x^2 + 9) dx$	
	$= \pi \left[\frac{1}{5}x^5 + 2x^3 + 9x \right]_0^2$	dM1 A1
	$= \frac{202}{5} \pi \text{ cm}^3 \quad *$	A1
		(4)
(b)	$\pi \int_0^2 x(x^2 + 3)^2 dx = \pi \int_0^2 (x^5 + 6x^3 + 9x) dx$	M1
	$= \pi \left[\frac{1}{6}x^6 + \frac{3}{2}x^4 + \frac{9}{2}x^2 \right]_0^2$	A1
	$= \frac{158}{3} \pi$ (Or by chain rule or substitution)	A1
	$\text{C of m} = \frac{158}{3} \times \frac{5}{202}, = 1.3036... = 1.30 \text{ cm}$	M1 A1
		(5)
(c)	Mass ratio $2 \times \frac{202}{5} \pi \quad \frac{1}{3} \pi \times 7^2 \times 6 \quad \left(\frac{404}{5} + 98 \right) \pi$	B1
	Dist from V $6.7 \quad 4.5 \quad \bar{x}$	B1
	$\frac{404}{5} \times 6.7 + 98 \times 4.5 = \left(\frac{404}{5} + 98 \right) \bar{x}$	M1 A1 ft
	$\bar{x} = \frac{\frac{404}{5} \times 6.7 + 98 \times 4.5}{\left(\frac{404}{5} + 98 \right)} = 5.494... = 5.5 \text{ cm} \quad \text{Accept 5.49 or better}$	A1
		(5)
(d)	$\tan \theta = \frac{6 - \bar{x}}{7} = \frac{0.5058...}{7}$	M1
	$\alpha = \tan^{-1} \left(\frac{6}{7} \right) - \tan^{-1} \left(\frac{0.5058...}{7} \right) = 36.468...^\circ = 36^\circ \text{ or better}$	M1 A1
		(3)
(17 marks)		
Notes:		
(a)		
M1: Using $\pi \int y^2 dx$ with the equation of the curve, no limits needed		

Question 6 notes *continued*

dM1: Integrating their expression for the volume.

A1: Correct integration inc limits now.

A1: Substituting the limits to obtain the GIVEN answer.

(b)

M1: Using $(\pi) \int xy^2 dx$ with the equation of the curve, no limits needed, π can be omitted.

A1: Correct integration, including limits; no substitution needed for this mark.

A1: Correct substitution of limits.

M1: Use of $\frac{\pi \int xy^2 dx}{\pi \int y^2 dx}$ with their $\pi \int xy^2 dx$. π must be seen in both numerator and denominator or in neither.

A1: **cs0.** Correct answer. Must be 1.30

(c)

B1: Correct mass ratio.

B1: Correct distances, from V or any other point, provided consistent.

M1: Attempting a moments equation.

A1ft: Correct equation, follow through their distances and mass ratio.

A1: Correct distance from V

(d)

M1: Attempting the tan of an appropriate angle, numbers either way up.

M1: Attempting to obtain the required angle.

A1: Correct final answer 2sf or more.